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WING DIVERGENCE AND ROLLING POWER

by

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SUMMARY

The fundamental static aeroelastic properties of a swept wing which has freedom to roll are studied. It is shown, using a simple example, that the divergence speed of a wing which can roll steadily is high or non-existent and consequently the rolling power of any aileron will vary almost linearly with dynamic pressure and, in particular, remains finite at the fixed-root divergence speed. Inboard ailerons have higher reversal speeds than outboard ailerons when the wing is swept back but the opposite holds true when the wing is swept forward.

Departmental Reference: Materials and Structures 221

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1 INTRODUCTION

During an investigation of the effect on aileron reversal of the coupling of flexure and torsion of swept wings through stiffness[1] it was realised that the properties found for the particular case of no stiffness coupling were not all that familiar. What was principally in question was the significance of divergence to rolling power at practical airspeeds. Rolling power - that is rolling velocity divided by airspeed for unit control deflection - becomes infinite at a divergence speed but the appropriate divergence speed is that for the case in which the rolling moment due to wing distortion is reacted by the airforces that arise from its rolling velocity rather than by the wing root being encastered as is the case for fixed-root divergence. Little attempt has been made previously[2-4] to estimate the speed at which this antisymmetric divergence occurs and so discover its practical significance. The purpose of the work reported here was to remedy this deficiency.

The previous investigation[1] did not include rolling power as a subject and that omission is rectified here for the case of wings with no stiffness coupling. Arbitrary modes describe the structural properties of the model as they are believed to be the most efficient and instructive form of structural representation in cases such as this. First the general equation for a rolling wing is described. From this equation the equations for fixed-root and antisymmetric divergence, *ie* divergence when the wing is free to roll, are derived. The rolling-power equation is also obtained. The rolling power can be evaluated by dividing one function of speed by another. The zeros of the dividend give the aileron reversal speeds and the divisor is zero at antisymmetric divergence speeds.

The equations are then applied to an example wing. This wing has uniform spanwise properties and is assumed to distort in sets of modes $1 - \cos(2m - 1)(\pi/2)\eta$ flexurally and $\sin(2m - 1)(\pi/2)\eta$ torsionally (η is a non-dimensional spanwise coordinate). The aerodynamic forces are based on strip theory and the assumptions that the aspect ratio is large and the sweep small. Airforces due to inboard and outboard ailerons are represented by the extremes of discrete forces acting at arbitrary chordwise positions at the root and the tip of the wing. Forces at the root neither roll nor distort the wing but the ratios between the different actions are finite and from them can be derived the mathematical limits of the behaviour when the aileron is inboard and its span tends to zero.

Initially the wing is taken to be unswept so that only torsional flexibility is of interest. The number of arbitrary modes included in the calculation is varied to find the minimum needed to give good approximations to the antisymmetric divergence and aileron reversal speeds. A swept wing is then considered, first with only flexural flexibility but finally with both flexural and torsional flexibility.

2 THE ROLLING EQUATION

2.1 Description of equation

The equation of a wing rolling with constant angular velocity can be written

$$\begin{bmatrix} C_{ss} + E_{ss}\upsilon^{-2} & b_{sr} \\ c'_{rs} & b_{rr} \end{bmatrix} \begin{bmatrix} q_s \\ p\ell/V \end{bmatrix} = \begin{bmatrix} c_{sc} \\ c_{rc} \end{bmatrix} , \tag{1}$$

where q_s is the vector of the generalised coordinates of a set of arbitrary fixed-root wing modes, C_{ss} and E_{ss} are square matrices of aerodynamic and structural stiffness coefficients and υ^2 is a scaled dynamic pressure. c'_{rs} is a row matrix of aerodynamic coefficients derived from the virtual work done in roll by the airforces due to the wing distortions. p is the rolling velocity of the wing, ℓ is a typical length and V is the airspeed. b_{sr} and b_{rr} are a column matrix and a scalar derived from the virtual work done by the airforces due to the rolling velocity of the wing in the distortion modes and in roll respectively. Similarly c_{sc} and c_{rc} are derived from the virtual work done by unit displacement of the ailerons. For a rigid wing the rolling power is given by

$$\frac{p_r\ell}{V} = \frac{c_{rc}}{b_{rr}} . \tag{2}$$

2.2 Fixed-root divergence

The fixed-root divergence speeds are given by the zeros of the determinant $|C_{ss} + E_{ss}\upsilon^{-2}|$. A wing, prone to divergence, probably has more than one divergence speed but a good approximation to the lowest can generally be obtained using only one arbitrary mode each of flexure and torsion, if the modes are chosen with care. The fundamentals of the modes chosen for the example wing whose coefficients are evaluated in the Appendix are $\zeta = 1 - \cos(\pi/2)\eta$ for flexure and

$\theta = \sin(\pi/2)\,\eta$ for torsion and $|C_{ss}| = 0$ if there is no more than one flexure mode, because $d\zeta_m/d\eta = (2m-1)(\pi/2)\,\theta_m$. Hence, if the flexural axis is the reference axis and the modes are not coupled by structural stiffness,

$$|C_{ss} + E_{ss}\upsilon^{-2}| = (c_{ff}e_{tt} + c_{tt}e_{ff})\upsilon^{-2} + e_{ff}e_{tt}\upsilon^{-4} = e_{ff}e_{tt}\upsilon^{-4}\left(1 - \frac{\upsilon^2}{\upsilon_{de}^2}\right) . \tag{3}$$

where υ_{de}^2 is the dynamic pressure at fixed-root divergence and is given by

$$\upsilon_{de}^{-2} = -\left(\frac{c_{ff}}{e_{ff}} + \frac{c_{tt}}{e_{tt}}\right). \tag{4}$$

Substituting the values of c_{ff}, c_{tt}, e_{ff} and e_{tt} given in the Appendix we have

$$\left(\frac{\pi}{2}\right)^2 \upsilon_{de}^{-2} = -\left(\left(\frac{2}{\pi}\right)^2 A \tan\Lambda\ (\widetilde{EI})^{-1} + \xi_a(\widetilde{GJ})^{-1}\right) \tag{5}$$

as has been found before[1].

From this we see that aerodynamic axes upwind of flexural axes (ξ_a negative) and sweep forward (Λ negative) lead to fixed-root divergence. The first of these conditions is satisfied subsonically by most wings but if they are swept back they do not suffer fixed-root divergence if

$$\tan\Lambda > -\left(\frac{\pi}{2}\right)^2 A^{-1}\xi_a \frac{EI}{GJ} . \tag{6}$$

2.3 Antisymmetric divergence

Fixed-root divergence is unlikely to be a physically-realisable phenomenon outside the wind tunnel. Large lift forces are implied in the symmetric case and these would lead to large normal accelerations were the wing part of a free-flying aircraft. It has been suggested[5] that the appropriate case is one in which the lift force is reacted by the aircraft inertia but it seems far more realistic to take the mode of free divergence to be one in which the incidence is of different sign inboard and outboard so that any incremental upwards lift is opposed by an equal downwards lift[6-8]. This gives a speed about twice that of fixed-root divergence but this does not mean that an aircraft can be flown safely at the fixed-root divergence speed for an aircraft of which the wing is a part becomes difficult to trim near it[7,8].

The antisymmetric divergence speeds are the speeds are which the coefficient matrix on the left-hand side of equation (1) is singular and at them both the distortions of the wing and its rolling velocity are indeterminate. The appropriate equation is

$$\begin{bmatrix} C_{ss} + E_{ss}\upsilon^{-2} & b_{sr} \\ c'_{rs} & b_{rr} \end{bmatrix} \begin{bmatrix} q_s \\ p\ell/V \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \tag{7}$$

The order of the equation can be reduced by eliminating either q_s or $p\ell/V$. In the first case the equation reduces to

$$\left[b_{rr} - c'_{rs}\left(C_{ss} + E_{ss}\upsilon^{-2}\right)^{-1} b_{sr}\right] \frac{p\ell}{V} = 0 \tag{8}$$

and in the second to

$$\left[C_{ss} - b_{sr}b_{rr}^{-1}c'_{rs} + E_{ss}\upsilon^{-2}\right] q_s = 0 . \tag{9}$$

Thus the divergence dynamic pressures are given by the eigenvalues of $-(C_{ss} - b_{sr}b_{rr}^{-1}c'_{rs})^{-1}E_{ss}$ which can be compared to the fixed-root case for which the eigenvalues are those of $-C_{ss}^{-1}E_{ss}$.

2.4 Rolling power

Equation (1) itself can be reduced by eliminating q_s or $p\ell/V$. Simply eliminating q_s gives

$$\left[b_{rr} - c'_{rs}\left(C_{ss} + E_{ss}\upsilon^{-2}\right)^{-1} b_{sr}\right] \frac{p\ell}{V} = c_{rc} - c'_{rs}\left(C_{ss} + E_{ss}\upsilon^{-2}\right)^{-1} c_{sc} . \tag{10}$$

Whilst eliminating $p\ell/V$, solving for q_s and back-substituting for $p\ell/V$ gives

$$\frac{b_{rr}p\ell}{V} = c_{rc} - c'_{rs}\left(C_{ss} - b_{sr}b_{rr}^{-1}c'_{rs} + E_{ss}\upsilon^{-2}\right)^{-1}\left(c_{sc} - b_{sr}b_{rr}^{-1}c_{rc}\right) . \tag{11}$$

Equation (11) shows clearly that the rolling power is indeterminate only at the antisymmetric divergence speeds and not at the fixed root divergence speeds and that, if the free divergence speeds are high, rolling power varies more-or-less linearly with dynamic pressure. The aileron reversal speed can be found by

equating the right-hand side of either equation (10) or equation (11) to zero but a more convenient and informative equation is obtained by rearranging equation (1) after putting p zero

$$\begin{bmatrix} C_{ss} + E_{ss}\upsilon^{-2} & c_{sc} \\ c'_{rs} & c_{rc} \end{bmatrix} \begin{bmatrix} q_s \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (12)$$

which can be reduced to

$$\left| C_{ss} - c_{sc}c_{rc}^{-1}c'_{rs} + E_{ss}\upsilon^{-2} \right| = 0 \qquad (13)$$

an eigenvalue problem.

3 UNSWEPT WING

3.1 Antisymmetric divergence

When the wing is unswept the only distortions of importance are those in torsion. The modes chosen for the example happen to be exactly those of fixed-root divergence, $(C_{ss} + E_{ss}\upsilon^{-2})$ is a diagonal matrix and the mth element on the diagonal is

$$\tfrac{1}{2}\left(1 - (2m - 1)^2\upsilon_d^2/\upsilon^2\right) , \qquad (14)$$

where υ_d^2 is the lowest of the fixed-root divergence dynamic pressures as before.

From equation (8), with the values of the coefficients taken from the Appendix, the antisymmetric divergence pressures are given by the solutions, υ_{da}^2, of

$$1 - 6\left(\frac{2}{\pi}\right)^4 \sum_{m=1}^{\infty} \frac{(2m - 1)^{-4}}{\left\{1 - (2m - 1)^2 \frac{\upsilon_d^2}{\upsilon_{da}^2}\right\}} = 0 \ . \qquad (15)$$

If the series is truncated to only one term the solution is a pressure of $69.13\upsilon_d^2$. This has been quoted previously[7] as an approximation to the free-divergence pressure but more than the fundamental fixed-root mode is needed for an adequate description of the wing's distortion. If two terms of the series are included the lower of the two solutions is $8.188\upsilon_d^2$ and this is a good

approximation to the true pressure of $8.183\upsilon_d^2$. The wing is rolling at divergence and Fig 1 shows the incidence induced by the roll velocity as well as that due to distortion of the wing. The net incidence resembles $\sin(3\pi/2)\eta$, the first overtone mode of fixed-root divergence. There is an infinity of divergence pressures with the pressures getting closer to $(2m + 1)^2\upsilon_d^2$ as the order m gets higher.

The wing is uniform in the present case, hence $\theta' \propto \int_\eta^1 L \, d\eta_1$, and the angle at the tip in any of the divergence modes is zero for

$$\theta_1 = \theta_0 + \int_0^1 \theta' d\eta \propto \int_0^1 d\eta \int_\eta^1 L(\eta_1)d\eta_1 = \int_0^1 \eta L(\eta)d\eta = 0$$

because the net rolling moment is zero.

3.2 Rolling power

The extremes of possible spanwise positions of the aileron are covered by taking as examples ailerons whose spans tend to zero (point ailerons) at the tip and at the root. When the values of coefficients derived in the Appendix are substituted in the right-hand side of equation (10), its zeros are given by

$$1 - 2\left(\frac{2}{\pi}\right)^2 \left(\frac{\xi_b}{\xi_a}\right) \sum_{m=1}^{\infty} \frac{(2m - 1)^{-2}}{\left\{1 - (2m - 1)^2 \frac{\upsilon_d^2}{\upsilon^2}\right\}} = 0 \tag{16}$$

for the tip aileron and

$$1 - 2\left(\frac{2}{\pi}\right)\left(\frac{\xi_b}{\xi_a}\right) \sum_{m=1}^{\infty} \frac{(-1)^{m+1}(2m - 1)^{-1}}{\left\{1 - (2m - 1)^2 \frac{\upsilon_d^2}{\upsilon^2}\right\}} = 0 \tag{17}$$

for the root aileron.

If only one term of the series is taken, equation (16) gives

$$\frac{\upsilon_d^2}{\upsilon_r^2} = 1 - 2\left(\frac{2}{\pi}\right)^2 \frac{\xi_b}{\xi_a} , \tag{18}$$

where υ_r^2 is the reversal pressure.

If another term is included a quadratic equation is obtained

$$9\,\frac{\upsilon_d^4}{\upsilon_r^4} - \left(10 - 7.38519\,\frac{\xi_b}{\xi_a}\right)\frac{\upsilon_d^2}{\upsilon_r^2} + 1 - 0.900633\,\frac{\xi_b}{\xi_a} = 0 \ . \qquad (19)$$

The solution of equation (19) can be put in the form

$$18\,\frac{\upsilon_d^2}{\upsilon_r^2} = 10 - 7.38519\,\frac{\xi_b}{\xi_a} \pm \sqrt{\left(8 - 7.38519\,\frac{\xi_b}{\xi_a}\right)^2 - 2.88202\,\frac{\xi_b}{\xi_a}}$$

$$\approx 10 - 7.38519\,\frac{\xi_b}{\xi_a} \pm \left(8 - 7.38519\,\frac{\xi_b}{\xi_a}\right) \times \left\{1 - 1.44101\left(\frac{\xi_b}{\xi_a}\right)\left(8 - 7.38519\,\frac{\xi_b}{\xi_a}\right)^{-2}\right\}$$

if

$$\left(8 - 7.38519\,\frac{\xi_b}{\xi_a}\right) \gg 2.88202\,\frac{\xi_b}{\xi_a} \ . \qquad (20)$$

The more interesting root is given by

$$\frac{\upsilon_d^2}{\upsilon_r^2} \approx 1 - 0.820577\left(\frac{\xi_b}{\xi_a}\right) - 0.0100070\left(\frac{\xi_b}{\xi_a}\right) \bigg/ \left(1 - 0.923149\,\frac{\xi_b}{\xi_a}\right) \ . \qquad (21)$$

For a subsonic wing and conventional aileron ξ_b/ξ_a is probably somewhere between −1 and 0 in value and so, in the present context, the simplest equation for the reversal dynamic pressure, equation (18), gives a good approximation as it agrees with the two-term formula, equation (21), when $\xi_b = 0$ and gives a pressure less than 1% smaller when $(\xi_b/\xi_a) = -1$. The reversal speed is below the fixed-root divergence speed if the centre of the lift due to aileron rotation is aft of the flexural axis.

Substituting the simplest approximations to the divergence and reversal speeds in equation (10), modified to give $\bar{p}$, the rolling power of the flexible wing as a fraction of that of the equivalent rigid wing (equation (2)), we get

$$\bar{p} = \left\{1 - \left(1 - 2\left(\frac{2}{\pi}\right)^2 \frac{\xi_b}{\xi_a}\right)\frac{\upsilon^2}{\upsilon_d^2}\right\} \bigg/ \left(1 - 0.122134\,\frac{\upsilon^2}{\upsilon_d^2}\right) \ . \qquad (22)$$

The ratio is almost linear with dynamic pressure up to the fixed-root divergence speed and beyond.

The reversal speed of the inboard aileron when only one term of the series is included is given by

$$\frac{\upsilon_d^2}{\upsilon_r^2} = 1 - 2\left(\frac{2}{\pi}\right)\frac{\xi_b}{\xi_a} \tag{23}$$

and when two terms are included an approximate lower speed is given by

$$\frac{\upsilon_d^2}{\upsilon_r^2} \approx 1 - 1.22608\,\frac{\xi_b}{\xi_a} + 0.047159\left(\frac{\xi_b}{\xi_a}\right)\Bigg/\left(1 - 1.37934\,\frac{\xi_b}{\xi_a}\right) . \tag{24}$$

The reversal pressure given by the one-term approximation is only about 3% lower than that given by the two-term approximation when $\xi_b/\xi_a = -1$ and the comments made on the tip-aileron case apply equally to the root aileron. Note that the reversal speed with inboard aileron is lower than that with an outboard aileron because although the lift of an inboard aileron is a poor source of wing distortion it is an even poorer source of rolling moment.

4 SWEPT WING

4.1 Antisymmetric divergence

It has been shown in section 2.2 that a wing with fixed root and only flexural flexibility diverges at some airspeed if the wing is swept forward but not if the wing is swept back. However if the wing is allowed to roll it does not diverge at any airspeed whatever the sweep. The matrix of aerodynamic stiffness coefficients for the flexural modes is not symmetric because the airforces depend on the deflection slopes in the modes rather than the deflections themselves. Hence the eigenvalues associated with equation (9) need not be real and indeed, if enough arbitrary modes are taken, all are purely imaginary. The eigenvalue with the lowest modulus is

$$\upsilon_{da}^2 = (\pm 40.75i)\left(\frac{\pi}{2}\right)^4 \widetilde{EI}(A \tan \Lambda)^{-1} \tag{25}$$

and approximations to the imaginary constants of the next smallest eigenvalues are $\pm 326i$ and $\pm 1100i$. The rolling power of a wing with only flexural flexibility is therefore never indeterminate. When only four modes are taken the dynamic pressure is only about ½% greater in modulus than the true divergence

pressure. Purely-imaginary eigenvalues are associated with real skew-symmetric matrices and this is consistent with the number of degrees of freedom needed for a good approximation being about twice the number needed when the eigenvalues are real. When considering the differences between the divergence pressures in the free and encastered wing cases it should be remembered that in the free case the airforces due to wing distortion are only reacted by the airforces due to rolling velocity which are at their largest at the wingtip in contrast to the fixed-root case in which the distortion airforces are only reacted at the root.

ϕ , the variable introduced to represent the relative amounts of flexural and torsional flexibility of the wing, is

$$\phi = \left(\frac{2}{\pi}\right)^2 \left(\frac{A \tan \Lambda}{EI}\right)\left(\frac{GJ}{\zeta_a}\right) \tag{26}$$

and takes some of the aerodynamic and geometric properties into account. It will be assumed that ζ_a is negative (υ_d^2 is positive) which is not unreasonable for a conventional wing in subsonic flow, and hence positive values of ϕ represent sweptforward wings. Consideration of positive as well as negative values of ζ_a might increase the risk of confusion without adding much to the understanding of what happens in practical cases.

Fig 2 shows the variation of the inverse of the antisymmetric divergence pressure with flexibility ratio. When the flexibility ratio is zero the divergence is purely torsional and it can be seen that the figure is limited to only the lowest four divergence roots. Flexural and torsional flexibility seem to be independent in effect and, with the eigenvalues identical when ϕ is infinite, whatever its sign, the curves are symmetric about the υ_d^2/υ^2 axis. It has already been noted that antisymmetric divergence has an insignificant effect on rolling power when the wing is unswept and it appears that the same is the case when the wing is swept.

4.2 Rolling power

Rolling power when there is only flexural flexibility is easier to represent than antisymmetric divergence. When there is a point aileron at the root, c_{fc} is null (see Appendix) and the reversal speeds are identical with the fixed-root divergence speeds of the wing (equation (13)). The two-mode approximation leads to

$$\begin{vmatrix} 1 + \kappa^2 \upsilon^{-2} & -1 \\ 3 & 1 + 81\kappa^2 \upsilon^{-2} \end{vmatrix} = 0 \ , \qquad (27)$$

where $\kappa^2 = (\pi/2)^4 \widetilde{EI}(A \tan \Lambda)^{-1}$.

From equation (27) the smaller reversal pressure is $-1.0406\kappa^2$, the numerical factor of which can be compared with -1 for the one-term approximation (equation (5)) and -1.03968 for the accurate solution. The sign of the factor means that only wings that are swept forward suffer reversal if the aileron is at the root.

For a tip aileron the two-mode approximation leads to

$$\begin{vmatrix} 1 - 4/\pi + \kappa^2 \upsilon^{-2} & -(1 - 4/(3\pi)) \\ 3 - 4/\pi & 1 + 4/(3\pi) + 81\kappa^2 \upsilon^{-2} \end{vmatrix} = 0 \ . \qquad (28)$$

From equation (28) the smaller reversal pressure is $4.5039\kappa^2$, the numerical factor of which can be compared with 3.66 for the one-term approximation and 4.50958 for the accurate solution. The sign of the factor means that only wings that are swept back suffer reversal if the aileron is at the tip.

Fig 3 shows the result of including torsional flexibility as well, for three particular values of ξ_b/ξ_a . Note that it is the inverse of reversal pressure that is plotted against flexibility ratio. Again low reversal speeds are associated with tip ailerons when the wing is swept back and root ailerons when the wing is swept forward.

This can be explained using simple physical considerations. Imagine an unswept wing with an aileron which has been rotated trailing-edge down. The aileron deflection results in an upward lift which is opposed by the lift due to the wing twist, which is a consequence of the aileron lift acting aft of the flexural axis, and by the lift due to rolling velocity. The net rolling and bending moments must be zero at the root when the wing reaches its equilibrium rolling velocity. Thus the distribution of bending moment over the span of the wing when there is a point aileron at the wing tip must be as in Fig 4a. The curve of the bending moment due to the wing airforces must be concave upwards for these airforces are one-signed but the curve of the bending moment due to the aileron airforces is a straight line. There is thus a sagging bending moment over the

wing which results in it bending tip up. Imagine the wing to be swept by rotation. If it is swept backward the lift due to bending is opposite in sign to the aileron lift and the rolling power is reduced, although there is some alleviation of this because the wing flexure 'blows off', but if the wing is swept forward, not only is the lift due to bending of the same sign as the aileron lift, but it is enhanced by the lift due to the extra wing distortion due to the wing's tendency to diverge in flexure. When the aileron is inboard the net loading of the wing leads to a hogging moment (Fig 4b) which has quite the opposite effect.

The dotted lines in Fig 3 labelled -R indicate the negative values of $\upsilon_d^2/\upsilon_r^2$ obtained when the aileron is at the root of a sweptback wing which is flexible in flexure. These were the only significant negative values obtained. If consideration is restricted to cases in which υ_d^2 is positive (ξ_a negative) then rolling power increases with speed, for this particular arrangement, for it varies nearly as $1 - \upsilon^2/\upsilon_r^2$. Remembering that, on a rigid wing, an inboard aileron provides less rolling power than an outboard aileron because its moment arm is shorter, there appears to be an opportunity of optimising the rolling power of a sweptback wing by positioning the aileron to give the best compromise between basic rolling power and reversal speed.

It can be seen from Fig 3 that the effects of flexural and torsional flexibility on reversal speed are almost independent of each other if the flexibility ratio is anything but large. Truncating equation (24) and using the exact value for the flexure-only case we get, for the case of the root aileron when the flexibility ratio is low,

$$\frac{\upsilon_d^2}{\upsilon_r^2} \approx 1 - 1.226\frac{\xi_b}{\xi_a} + 0.962\phi \qquad (29)$$

remembering that ϕ is positive when the wing is swept forward; and similarly in the case of a tip aileron

$$\frac{\upsilon_d^2}{\upsilon_r^2} \approx 1 - 0.821\frac{\xi_b}{\xi_a} - 0.222\phi \ . \qquad (30)$$

That part of Fig 3c that relates to a root aileron duplicates the variation of fixed-root divergence speed with scaled flexibility ratio for, with ξ_b/ξ_a zero, both c_{fc} and c_{tc} are null.

Most of the curves were found from four flexural and four torsional modes as data but checks were made using eight of each type of mode. The values calculated with the lesser number of modes are good approximations to the lower reversal speeds, the more important, but occasionally are not as good for the upper speeds, of which there are more than those shown in the Figure. However they are always accurate enough for the characteristics of the curves to be correct.

4.3 Tailerons

Roll control is obtained on some aircraft by fitting them with tailplanes whose two sides can be rotated differentially in pitch (tailerons). In this case the airforces on the wing are not influenced directly by the control movements and the appropriate rolling equation is equation (1) with c_{sc} null and c_{rc} representing the rolling moment transmitted through the fuselage from the taileron. The fuselage and taileron are taken to be rigid in themselves. If c_{sc} is made null in equation (11) the rolling power of the taileron is obtained as

$$\frac{b_{rr}p\ell}{V} = \left[1 + c'_{rs}(C_{ss} - b_{sr}b_{rr}^{-1}c'_{rs} + E_{ss}\upsilon^{-2})^{-1}b_{sr}b_{rr}^{-1}\right]c_{rc} \qquad (31)$$

and the free divergence speed is again the speed at which the rolling power is indeterminate. However the fixed-root divergence speed is also significant in this case for if c_{sc} is put null in the aileron-reversal equation, equation (13), it becomes the same as the fixed-root divergence equation, *ie*

$$|C_{ss} + E_{ss}\upsilon_r^{-2}| = 0 \quad . \qquad (32)$$

Thus the variation of aileron-reversal pressure with flexibility ratio is identical to that for a root aileron whose lift acts at the stiffness axes of the wing and is given by the lines marked R in Fig 3c and rolling power can increase with speed if the wing is flexurally flexible and swept back. In any case the variation of rolling power with speed will be near linear at flight speeds as before.

5 CONCLUDING REMARKS

A study of the rolling equation of a wing has shown that there is a free antisymmetric divergence case analogous to the free symmetric divergence case. When symmetric divergence occurs the distortion mode is such that the net lift on the wing is zero and when antisymmetric divergence occurs the mode is such that

the net rolling moment is zero. In the symmetric case the wing pitches by an amount sufficient to equalise the upwards and downwards lifts on it and in the antisymmetric case the wing rolls at a velocity sufficient to equalise its positive and negative rolling moments. In both cases the divergence mode is overtone in shape and hence the critical speeds are much higher than that of fixed-root divergence. The order of the overtone is the higher in the antisymmetric case and so the divergence speed is also the higher.

Furthermore only wings with little or no sweepback have real free antisymmetric divergence speeds. The eigenvalues corresponding to the divergence dynamic pressures of wings with only flexural flexibility are peculiar in that they are imaginary numbers. This means that such wings do not diverge and the eigenvalues are seen to be so large that, at practical airspeeds, rolling power is little affected by possible divergence-type distortion of the wing. When torsional flexibility is included the dominant eigenvalues change first to conjugate complex numbers and so the wing still does not diverge. The behaviour of the wing is identical for equal amounts of sweepback and sweepforward (Fig 2) and this is consistent with the aerodynamic coefficients for lift due to torsion having the same values for equal amounts of sweepback and sweepforward and the coefficients for lift due to flexure being equal but opposite in sign in the same circumstances.

It is the free antisymmetric divergence speed and not the fixed-root speed that could be of significance in the calculation of rolling power but as it is high or non-existent it is of little consequence in practice. If the airforces are linear, rolling power is very close to being a linear function of dynamic pressure being zero at the aileron reversal pressure. This reversal pressure depends on the spanwise position of the aileron as well as the sign of the sweep of the wing. It is well known that a wingtip aileron on a swept back wing has a low reversal pressure but the reversal pressure for an inboard aileron on the same wing can be negative which means that rolling power increases with dynamic pressure although its absolute value might be small owing to the shorter moment arm.

The calculations demonstrate the economy that can be achieved when well chosen but simple arbitrary modes are used for the description of structural distortion in studies such as this. In nearly all instances two such modes of each type have been shown to be enough to give results sufficiently accurate for practical purposes.

Appendix

EVALUATION OF THE COEFFICIENTS FOR AN EXAMPLE WING

A.1 Example wing

The example wing is of uniform chord c perpendicular to the stiffness axis which is of length ℓ and sweepback Λ. Air, of density ρ, is flowing past the wing at speed V. Let the downward heave of the stiffness axis in the flexural mode (q_{fm}) be $\zeta_m c$, where $\zeta_m = 1 - \cos(m - \frac{1}{2})\pi\eta$ and η is zero at the wing root and unity at the tip, and the nose-up incidence (sections perpendicular to the stiffness axis) in the torsion mode (q_{tm}) be θ_m, where $\theta_m = \sin(m - \frac{1}{2})\pi\eta$. The mode appropriate to roll (q_r) is rotation about the axis in the direction of the line of flight which passes through the wing root. Let the generalised coordinate associated with aileron angle, taken as constant along the aileron span and positive trailing edge down, be q_c.

A.2 Aerodynamic coefficients

The work done in a small displacement of the wing is

$$W = -\gamma \int_{\text{wing}} L(c\zeta\delta q_f + \ell\eta \cos\Lambda\, \delta q_r) + \int_{\text{wing}} M(\theta\delta q_t - \sin\Lambda\, \delta q_r)$$

$$= \rho c^2 \ell V^2 \cos\Lambda\, a \begin{bmatrix}\delta q_f' & \delta q_t' & \delta q_r'\end{bmatrix} \begin{bmatrix} C_{ff} & C_{ft} & b_{fr} \\ C_{tf} & C_{tt} & b_{tr} \\ c_{rf}' & c_{rt}' & b_{rr} \end{bmatrix} \begin{bmatrix} q_f \\ q_t \\ (p\ell/V)q_r \end{bmatrix} , \qquad \text{(A-1)}$$

where, following Yates[9],

$$L\ \text{(upward)} = -\rho c\ell V^2 \cos^2\Lambda\, a\{\gamma\zeta' q_f + \theta q_t + (p\ell/V)\cos\Lambda\, \eta q_r + \ell_\beta q_c\} , \qquad \text{(A-2)}$$

$$M\ \text{(nose up)} = \rho c^2 \ell V^2 \cos^2\Lambda\, a\, \{\xi_a(\gamma\zeta' q_f + \theta q_t + (p\ell/V)\cos\Lambda\, \eta q_r) + \xi_b \ell_\beta q_c\}, \qquad \text{(A-3)}$$

where a is the $dC_L/d\alpha$ of the wing, ℓ_β multiplied by a is dC_L/dq_c and ξ_a and ξ_b are the chordwise distances of the wing and aileron lift respectively, aft of the stiffness axis and scaled by the chord. $\zeta' = d\zeta/d\eta$ and $\gamma = A^{-1} \tan\Lambda$ where $A = \ell/c$ and it has been assumed that the value of γ is low, *ie* the aspect ratio is high and the sweep is low, and so terms which have γ as a factor are negligible compared with unity.

Note that with the modes assumed $c_{p,fm} = (2m - 1)(\pi/2)\gamma c_{p,tm}$ when $p = fm,\ tm,\ r$.

The aerodynamic coefficients are

$$c_{fm,fn} = \gamma \int_0^1 \zeta_m \zeta_n' d\eta = (2n - 1)(\pi/2)\gamma \int_0^1 (1 - \cos(m - \tfrac{1}{2})\pi\eta \sin(n - \tfrac{1}{2})\pi\eta\, d\eta$$

$$= \gamma\left(1 - \frac{1}{2}\frac{2n - 1}{m + n - 1}\right), \quad m + n \text{ even}; \quad = \gamma\left(1 - \frac{1}{2}\frac{2n - 1}{n - m}\right), \quad m + n \text{ odd},$$

and, from integration by parts, $c_{fm,fn} + c_{fn,fm} = \gamma$,

$$c_{tm,tn} = \xi_a \int_0^1 \theta_m \theta_n d\eta = \xi_a \int_0^1 \sin(m - \tfrac{1}{2})\pi\eta \sin(n - \tfrac{1}{2})\pi\eta d\eta = \tfrac{1}{2}\xi_a, \quad m = n$$

$$= 0, \quad m \neq n$$

......(A-4)

$$c_{r,fm} = \gamma A \cos\Lambda \int_0^1 \zeta_m' \eta d\eta = (2m - 1)(\pi/2)\gamma A \cos\Lambda \int_0^1 \eta \sin(m - \tfrac{1}{2})\pi\eta d\eta$$

$$= \gamma A \cos\Lambda \frac{(-1)^{m+1}}{2m - 1}(2/\pi),$$

$$b_{fm,r} = A \cos\Lambda \int_0^1 \zeta_m \eta d\eta = A \cos\Lambda \int_0^1 \eta(1 - \cos(m - \tfrac{1}{2})\pi\eta) d\eta$$

$$= A \cos\Lambda \left(\frac{1}{2} + \frac{(-1)^m}{2m - 1}(2/\pi) + \frac{1}{(2m - 1)^2}(2/\pi)^2\right),$$

$$b_{tm,r} = \xi_a A \cos\Lambda \int_0^1 \theta_m \eta d\eta = \xi_a A \cos\Lambda \int_0^1 \eta \sin(m - \tfrac{1}{2})\pi\eta d\eta$$

$$= \xi_a A \cos\Lambda \frac{(-1)^{m+1}}{(2m - 1)^2}(2/\pi)^2,$$

$$b_{rr} = (A \cos\Lambda)^2 \int_0^1 \eta^2 d\eta = \frac{1}{3}(A \cos\Lambda)^2 .$$

The ailerons considered can be thought of as point ailerons because their spanwise dimension is taken to be almost zero. They are taken as located at either the root or the tip of the wing. Only ratios of the coefficients are required.

$$A\cos\Lambda\, c_{fm,c}/c_{rc} = (1-\cos(m-\tfrac{1}{2})\pi\eta)/\eta \to 0 \;, \quad \eta \to 0$$
$$= 1 \;, \quad \eta = 1 \qquad , \quad \text{(A-5)}$$

$$A\cos\Lambda\, c_{tm,c}/c_{rc} = \xi_b \sin(m-\tfrac{1}{2})\pi\eta/\eta \to (2m-1)(\pi/2)\xi_b \;, \quad \eta \to 0$$
$$= (-1)^{m+1}\xi_b \;, \quad \eta = 1 \;.$$

A.3 Structural coefficients

Twice the strain energy of the wing in an arbitrary distortion is

$$2V = (A^2\ell)^{-1}EI\int_0^1 \zeta''^2 q_f^2\, d\eta + \ell^{-1}GJ\int_0^1 \theta'^2 q_t^2\, d\eta = \rho c^2 \ell V_0^2 a \lfloor q_f' \; q_t' \rfloor \lceil E_{ff} \; E_{tt} \rfloor \{q_f \; q_t\} \quad \text{(say)}, \qquad \text{......(A-6)}$$

where V_0 is a reference speed.

$$\rho c^2 \ell V_0^2 a e_{fm,fn} = (A^2\ell)^{-1}EI\int_0^1 \zeta_m''\zeta_n''\, d\eta$$
$$= (2m-1)^2(2n-1)^2(\pi/2)^4(A^2\ell)^{-1}EI\int_0^1 \cos(m-\tfrac{1}{2})\pi\eta\,\cos(n-\tfrac{1}{2})\pi\eta\, d\eta$$
$$= \tfrac{1}{2}(2m-1)^4(\pi/2)^4(A^2\ell)^{-1}EI \;, \quad m = n \;; \quad = 0, \; m \neq n \;. \qquad \text{(A-7)}$$

Similarly

$$\rho c^2 \ell V_0^2 a e_{tm,tn} = \ell^{-1}GJ\int_0^1 \theta_m'\theta_n'\, d\eta = \tfrac{1}{2}(2m-1)^2(\pi/2)^2\ell^{-1}GJ \;, \quad m = n \;; \; = 0, \; m \neq n \;.$$

E_{ss} is a diagonal matrix,

$$\lceil E_{ff}, E_{tt} \rfloor = \tfrac{1}{2}\left\lceil \left((2m-1)\pi/2\right)^4 \widetilde{EI}/A^2, \left((2m-1)\pi/2\right)^2 \widetilde{GJ} \right\rfloor \quad , \tag{A-8}$$

where $EI = \rho c^2 \ell^2 V_0^2 a \widetilde{EI}$, etc.

The scaled airspeed is obtained from the relationship between the different scaling factors used for the aerodynamic and structural stiffness coefficients (equations (A-3) and (A-7)) as

$$\upsilon = \frac{V \cos \Lambda}{V_0} \tag{A-9}$$

and a typical form for the divergence equation is

$$|C + E\upsilon^{-2}| = 0 \quad . \tag{A-10}$$

LIST OF SYMBOLS

A	ℓ/c
a	$dC_L/d\alpha$ for wing
b_{sr}, b_{rr}	coefficients of work done in wing distortion modes and roll by airforces due to rolling velocity
c	wing chord perpendicular to the stiffness axis
c_{sc}, c_{rc}	coefficients of work done in wing distortion modes and roll by airforces due to aileron
c'_{rs}	coefficients of work done in roll by airforces due to wing distortion
C_{ss}	square matrix of coefficients of work done in wing distortion modes by airforces due to wing distortion
E_{ss}	square matrix of structural stiffness coefficients
$EI, \widetilde{EI}$	flexural rigidity of wing, lift scaled
$GJ, \widetilde{GJ}$	torsional rigidity of wing, lift scaled
L	lift
ℓ	length of stiffness axis
$\ell_\beta a$	dC_L/dq_c for aileron
M	moment
p, p_r	rolling velocities of flexible and rigid wings
$\bar{p}$	scaled rolling velocity, p/p_r
q_c	generalised coordinate of aileron angle
q_r	generalised coordinate of roll of aircraft
q_s	generalised coordinates of wing distortion modes, $\{q_f, q_t\}$
V	airspeed
γ	$A^{-1} \tan \Lambda$
ζ	scaled flexural displacement in distortion modes (q_f)
η	scaled spanwise coordinate of wing
θ	torsional displacement in distortion modes (q_t)
κ^2	scaled flexural rigidity, equation (27) *et seq*
Λ	wing sweepback
ξ_a	distance aerodynamic centre of wing is aft of wing stiffness axis as a fraction of wing chord
ξ_b	distance centre of aileron lift is aft of wing stiffness axis as a fraction of wing chord
ρ	air density
ν^2	dynamic pressure

LIST OF SYMBOLS (concluded)

ν_d^2, ν_{de}^2 dynamic pressure at fixed-root divergence, torsional flexibility only and both flexure and torsion

ν_{da}^2, ν_r^2 dynamic pressure at antisymmetric divergence, aileron reversal

ψ scaled flexibility ratio,

$$\left(\frac{2}{\pi}\right)^2 \left(\frac{A \tan \Lambda}{EI}\right) \left(\frac{GJ}{\xi_a}\right)$$

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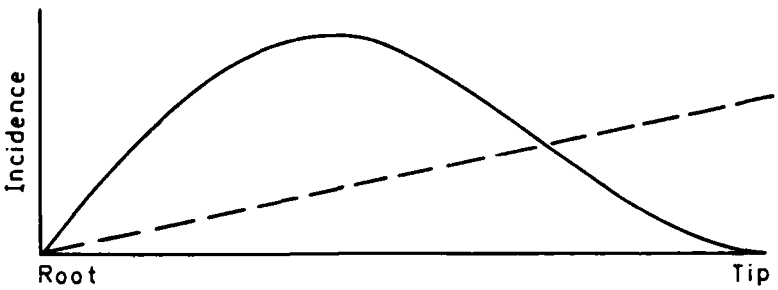


Fig 1 Torsional antisymmetric divergence mode

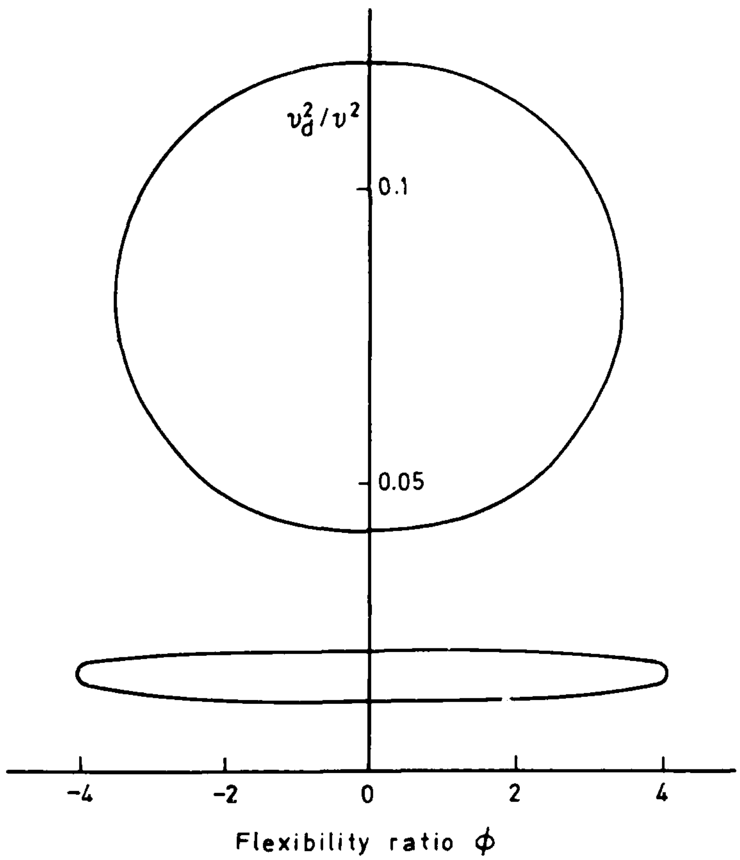
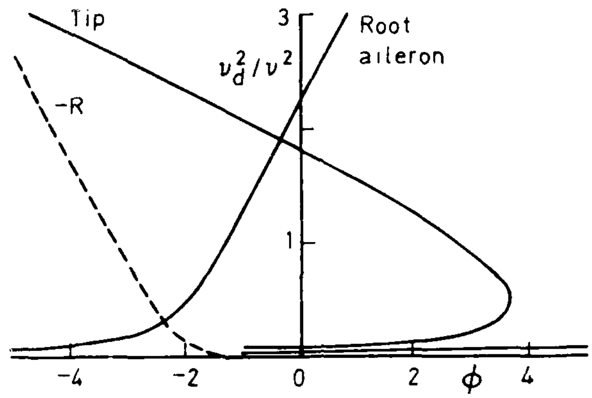
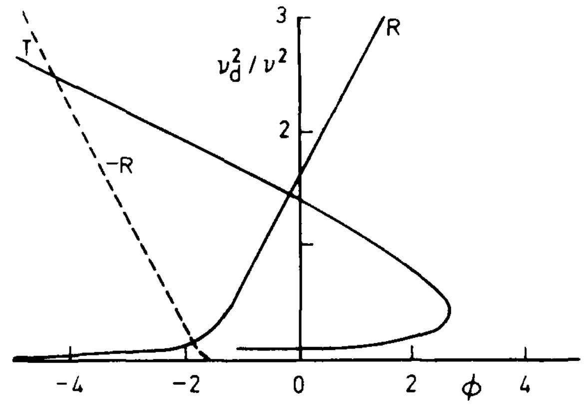


Fig 2 Inverse of antisymmetric-divergence pressure

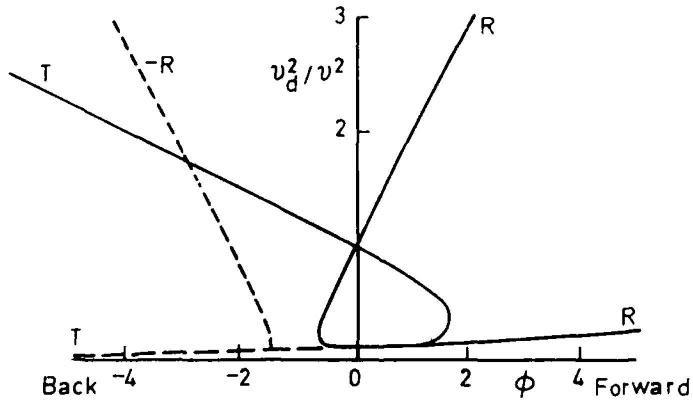
Fig 3



a) $\xi_b/\xi_a = -1$



b) $\xi_b/\xi_a = -\frac{1}{2}$



c) $\xi_b/\xi_a = 0$

Fig 3 Inverse of reversal pressure v flexibility ratio

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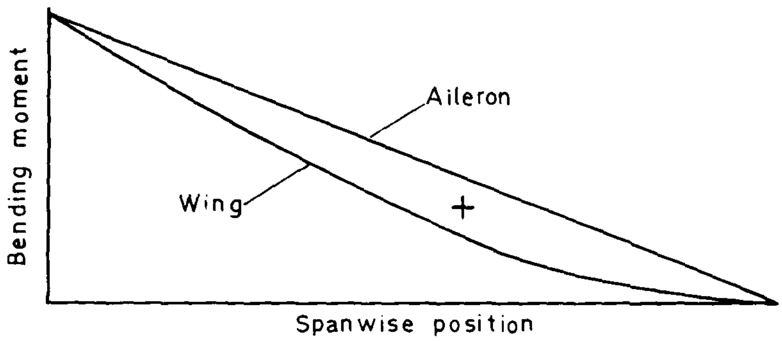
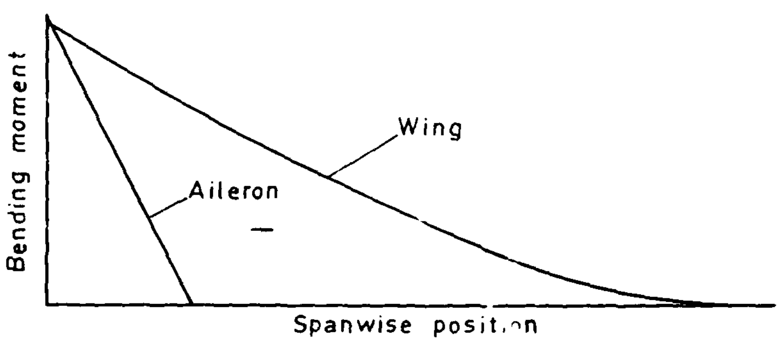


a) Tip aileron



b) Inboard aileron

Fig 4 Net bending moments on wing

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Aeroelasticity, Rolling power, Divergence, Great Britain. (SDW)

17\. Abstract

The fundamental static aeroelastic properties of a swept wing which has freedom to roll are studied. It is shown, using a simple example, that the divergence speed of a wing which can roll steadily is high or non-existent and consequently the rolling power of any aileron will vary almost linearly with dynamic pressure and, in particular, remains finite at the fixed-root divergence speed. Inboard ailerons have higher reversal speeds than outboard ailerons when the wing is swept back but the opposite holds true when the wing is swept forward.

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